Towards a Method for the Assessment of Cerebral Arteriovenous Malformations Surgery with a Bi-Directional Doppler System for Blood Flow Measurement

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ABSTRACT:
This paper shows the application of Doppler flow measurement techniques to assess the development of surgical treatment of cerebral arteriovenous malformations during the intervention. Also, it is shown the architecture of a Bi-directional Doppler System for Blood Flow Measurement that has already been used to evaluate the bypass quality in coronary revascularization surgeries. Four methods of spectral analysis are described, which include the analysis of the real Doppler signal, its analytic signal and its quadrature signal, using Fourier transform, Gabor transform and time-frequency distributions of the Cohen class, as well as algorithms of flow separation.

KEYWORDS:

1. Introduction
This paper shows the application of Doppler flow measurement techniques to assess the development of surgical treatment of cerebral arteriovenous malformations during the intervention.

The Bidirectional Doppler System for Blood Flow Measurement shown in this paper has already been successfully used to evaluate the bypass quality in coronary revascularization surgeries. [22][23][24][25][26][27][28][29]. Figure 18 shows a photograph of the prototype of the system used during surgeries. Figure 19 shows a photograph of the system screen assessing a coronary bypass.

It is now intended to use this system to assess the development of surgeries for removal of cerebral arteriovenous malformations.

2.- Arteriovenous Malformations (AVM)
An arteriovenous malformation (AVM) is an abnormal defective formation of blood vessels that drains arterial blood directly into the veins without passing through capillaries [1]. Consequently, the partial lack of blood flow with oxygen in the capillaries can cause tissue damage in the affected areas. This phenomenon of partial lack of blood supply to the tissues caused by an AVM is known as vascular thievery [2] [3] [4].

Figure 1 shows a diagram of a healthy network of capillaries, which distributes oxygenated arterial blood to the surrounding tissues adequately. Figure 2 shows an outline of an arteriovenous malformation that drains oxygenated arterial blood directly into the veins, which causes a significant decrease in the supply of oxygenated arterial blood to the network of capillaries, degrading their function.

Other treatments against AVM include conservative treatments (non-invasive treatments) and surgical treatments (invasive treatments). The latter include surgical removal (resection), endovascular embolization and stereotactic radiosurgery[5].

This paper shows the application of Doppler flowmetry techniques to evaluate the development of surgical treatment during the intervention. The pre-intervention blood flow condition may be as follows: there is a stolen high flow in both the AVM feeding artery and the vein that drains the AVM (points 2 and 3 in figure 2), and an abnormal low flow in the artery that feeds healthy capillaries (point 1 in Figure 2). During the procedure, the stolen blood flow through the artery feeding the AVM is partially or totally blocked (point 2 in figure 2). The post-intervention blood flow condition is low flow (or non-flow) in both the AVM feeding artery and the AVM draining vein (points 2 and 3 in Figure 2), and a restored high flow in the artery feeding the healthy capillaries (point 1 in Figure 2).

Figure 1: Network of healthy capillaries.
3.- Bidirectional Doppler System for Measurement of Blood Flow

The basis of the Doppler Flow measurement applied to blood flow is that the instantaneous mean frequency of the Doppler signal is proportional to the instantaneous mean velocity of the blood flow through the artery or vein [6] [7].

\[ f_{\text{Doppler}} = \frac{2f_0 \cos(\alpha)}{c} V_{\text{BloodFlow}} \]  (1)

Hence, one of the main objectives of the Bi-Directional Doppler System for Blood Flow Measurement is to estimate the instantaneous mean frequency of the Doppler signal.

The Bi-Directional Doppler System for Blood Flow Measurement consists of three modules. See figure 3. The first module is hardware. This basically consists of the blood flow detector probe, the transducer and the electronic devices that deliver a Quadrature Doppler signal in (two channels: Doppler signal -D signal- and Doppler signal in Quadrature -Q signal-).

The second module is software. This basically consists of a collection of spectral analysis programs whose purpose is to contribute to the estimation of the spectrogram and the instantaneous mean frequency of the Doppler signal.

The third module basically consists of the adequate graphical display of the results produced in the spectral estimation module.

4.- Spectral Analysis

Using hardware, the Bi-Directional Doppler System for Blood Flow Measurement generates two input signals to be analyzed by the spectral estimation module: the first is a Doppler signal (D), and the second is another Doppler signal but in Quadrature (Q) with respect to D signal.

These signals are represented by time series of real numbers sampled according to a certain sampling frequency, and their spectral analysis is performed by processing a succession of consecutive data windows, of a certain length, with or without overlap. As a result of processing the succession of data windows, a spectrogram is obtained from which the instantaneous mean frequency is estimated. See figure 4.

There are different options for spectrally analyzing the Doppler D and Q signals. Four of these are explored in this paper.

The four options that will be explained are: the use of Short Time Fourier Transform (STFT) or Gabor Transform to analyze the Doppler signal, the use of Time Frequency Distributions to analyze the analytic signal corresponding to the Doppler signal, the use of Flow Separation Algorithms to analyze the Quadrature Doppler signal, and the use of Time Frequency Distributions to analyze the Quadrature Doppler signal.

4.a.- STFT or Gabor Transform

In this method unidirectional flow is assumed (otherwise, if there is bidirectional flow, this option is not adequate). Each data window corresponding to the D signal (real signal) is analyzed consecutively. First, its STFT [8] or its Gabor transform [9] is calculated. Then, its spectrogram is calculated. Finally, as a result, the instantaneous mean frequency is calculated. See Figure 5.

The Short Time Fourier Transform (STFT) of a signal \( x(t) \) is defined as:
\[
STFT \{ x(t) \} = \hat{X}_{STFT}(t, \omega) = \int_{-\infty}^{\infty} W(\tau - t) x(\tau) e^{-j\omega \tau} d\tau \tag{2}
\]

where \( W(t) \) is a window function; for example: rectangular, Hanning, Kaiser, etc.

The Gabor Transform of a signal \( x(t) \) is defined as:
\[
G_{\alpha} \{ x(t) \} = \hat{X}_{G\alpha}(t, \omega) = \int_{-\infty}^{\infty} e^{-\pi \alpha (\tau - t)^2} x(\tau) e^{-j\omega \tau} d\tau \tag{3}
\]

where \( \alpha \) is a parameter that optimizes the tradeoff between time-frequency resolution. Comparing (1) with (2), it is observed that \( W(t) = e^{-\pi \alpha t^2} \) is a window function of Gaussian type.

For the transformations (2) and (3), the spectrogram is defined respectively as:
\[
\text{spec}(t, \omega) = \left| \hat{X}_{STFT}(t, \omega) \right|^2 \tag{4}
\]
\[
\text{spec}(t, \omega) = \left| \hat{X}_{G\alpha}(t, \omega) \right|^2 \tag{5}
\]

Finally, for this method, the instantaneous mean frequency is defined as:
\[
f_i(t) = \frac{\int_{0}^{\infty} \omega \cdot \text{spec}(t, \omega) d\omega}{\int_{0}^{\infty} \text{spec}(t, \omega) d\omega} \tag{6}
\]

4.b.- Analytic Signal with Time-Frequency Distributions
Also, unidirectional flow is assumed in this method. Each data window that corresponds only to D signal (a real signal) is analyzed consecutively, ignoring the Q signal. First, its analytic signal which is a complex signal, is calculated. The analytic signal calculation can be made in time domain, or in frequency domain. In time domain, it is calculated by the Hilbert transform of the D signal (using a convolution); while in frequency domain, its spectrum is constructed using the Fourier transform of the D signal. Then some time-frequency distribution of Cohen’s class \([10][11]\) of the analytical signal is calculated (it may also be the spectrogram). Finally, as a result, the instantaneous mean frequency is calculated. See Figure 6.

The analytic signal calculation in time domain is described below. The analytic signal of a real signal \( x_r(t) \) is defined as:
\[
x_a(t) = x_r(t) + jH \{ x_r(t) \} \tag{7}
\]
where the H operator means the Hilbert transform. The Hilbert Transform of a signal \( x(t) \) is defined as:
\[
H \{ x(t) \} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau \tag{8}
\]

Note that the Hilbert transform can be calculated by convolution:
\[
H \{ x(t) \} = h(t) * x(t) \tag{9}
\]

where \( h(t) = 1/(\pi t) \).

Now, the analytic signal calculation in frequency domain is described below. The Fourier transform of a signal \( x(t) \) is defined as:
\[
F \{ x(t) \} = \hat{X}(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{10}
\]

and the Inverse Fourier Transform of \( \hat{X}(\omega) \) is defined as:
\[
F^{-1} \{ \hat{X}(\omega) \} = x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{X}(\omega) e^{j\omega t} d\omega \tag{11}
\]

Then, let \( \hat{X}_r(\omega) \) be the Fourier transform of a real signal \( x_r(t) \). The Fourier transform of the analytic signal is constructed as:
\[ \hat{X}_a(\omega) = \hat{X}_r(\omega) + \text{sign}(\omega)\hat{X}_r(\omega) = \begin{cases} 
2\hat{X}_r(\omega), & \omega > 0 \\
\hat{X}_r(\omega), & \omega = 0 \\
0, & \omega < 0 
\end{cases} \quad (12) \]

where \(\text{sign}(\omega)\) is the sign function. The analytic signal \(x_a(t)\) is the inverse Fourier transform of \(\hat{X}_a(\omega)\).

On the other hand, the Time-Frequency Distributions (TFD) of the Cohen Class are defined as:

\[ TFD_\phi(t,\omega) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( \mu + \frac{T}{2} \right) x^*(\mu - \frac{T}{2}) \phi(\theta,\tau)e^{-j\theta t + j\mu\tau} d\theta d\mu d\tau \quad (13) \]

where \(\phi(\theta,\tau)\) is the distribution kernel, which determines its properties. Table 1 shows the kernel of some of the major TFD.

Finally, for this method, the instantaneous mean frequency \([14][15]\) is defined as:

\[ f_i(t) = \frac{\int_0^\infty \omega \cdot TFD_\phi(t,\omega) d\omega}{\int_0^\infty TFD_\phi(t,\omega) d\omega} \quad (14) \]

4.c.- Quadrature Signal with Flow Separation Algorithm

This method assumes bidirectional flow (note that unidirectional flow is a particular case). Each data window corresponding to the quadrature signal \(D(t) + jQ(t)\), which is a complex signal, is analyzed consecutively. First, a flow separation algorithm \([12][13]\) is applied to generate two real signals: one forward flow signal \(x_{\text{forward}}(t)\) and one reverse flow signal \(x_{\text{reverse}}(t)\). The flow separation algorithm involves calculating the Hilbert transform of the D signal. Finally, each unidirectional flow signal \(x_{\text{forward}}(t)\) and \(x_{\text{reverse}}(t)\) is analyzed separately according to the first or second options set forth above. See Figure 7.

The flow separation algorithm is:

\[ x_{\text{forward}}(t) = Q(t) - H\{D(t)\} \quad (15) \]

\[ x_{\text{reverse}}(t) = Q(t) + H\{D(t)\} \quad (16) \]

Figure 7. Spectral estimation for bidirectional flow using the quadrature Doppler signal (complex signal) and a flow separation algorithm. The result of this algorithm are two
real signals, one of forward flow (F) and one of reverse flow (R), whose spectral analyzes are done separately.

**4.d.- Quadrature Signal with Time-Frequency Distributions**

This method also assumes bidirectional flow. Each data window corresponding to the quadrature signal 
\( D(t) + jQ(t) \), which is a complex signal, is analyzed consecutively. First, some time-frequency distribution of the Cohen class of the quadrature signal (13) is calculated (it can also be the spectrogram). Finally, as a result, the instantaneous mean frequency is calculated. See figure 8.

For this method, the instantaneous mean frequency is defined as:

\[
f_i(t) = \frac{\int_{-\infty}^{\infty} \omega \cdot TFD_{\omega}(t, \omega) d\omega}{\int_{-\infty}^{\infty} TFD_{\omega}(t, \omega) d\omega}
\]  

(17)

Figure 8. Spectral estimation for bidirectional flow using the quadrature Doppler signal (complex signal) and the time-frequency distributions of the Cohen class.

**5.- Results**

The results are presented in two sections. The first section consists of a qualitative comparison of the methods. The second section consists of a preliminary assessment of a surgical procedure to remove a cerebral arteriovenous malformation.

The amplitude of the spectrograms is plotted on a logarithmic scale, normalized with the maximum expected power in each case. A range of 0 [dB] to -8 [dB] is plotted.

Note that the nature of the flow signal being analyzed is unidirectional because they correspond to flow signals in cerebral arteries. Conventionally it is said to be a forward flow signals, which have no reverse flow components. When a complex flow signal (such as the analytic signal or the quadrature signal) is spectrally analyzed, the positive frequency components are conventionally associated with the forward flow and the negative frequency components with the reverse flow.

**5.a.- Qualitative comparison of methods**

This section only shows results of the spectral analysis of the artery that feeds the capillaries with its restored flow. See figure 15.

Figure 9 shows the spectrogram of the signal D (real signal) processed with STFT. Figure 10 shows the spectrogram of the D signal (real signal) processed with Gabor transform (method 4.a). Since a real signal is analyzed in both cases, the unidirectional flow requirement is requested. If the flow were bidirectional, the information corresponding to the forward and reverse flow would be inseparably mixed in the real signal. Also, since a real signal is analyzed, the spectrogram is symmetrical. So the same information contained in the positive frequencies is in the negative frequencies of the spectrogram. For this reason, the integral to calculate the instantaneous mean frequency (6) is over the interval \([0,\pi]\). Negative frequencies are thus ignored.

The spectrogram of the analytic signal of the D signal processed with time frequency distributions (method 4.b) is shown in Figure 11. The analytic signal is a complex signal. Since the spectrogram of a complex signal is calculated, the spectrogram is not symmetrical. So the information contained in the positive and negative frequencies of the spectrogram is different. However, since the original signal being analyzed is the D signal, which is a real signal, the unidirectional flow requirement is requested. Consequently, only the positive frequencies have information and the negative frequencies do not, canceling out. For this reason, the integral to calculate the instantaneous mean frequency (14) is over the interval \([0,\pi]\).

Figure 12 shows the spectrograms of forward and reverse flow signals processed with STFT. The flow separation algorithm is applied to the quadrature Doppler signal (D+jQ), which is a complex signal. So the information corresponding to the forward and reverse flow is mixed but separable in the complex signal. However, since the nature of the flow signal being analyzed is unidirectional, only the spectrogram associated with the forward flow has information, whereas the spectrogram associated with the reverse flow does not, canceling out.

Figure 13 shows the spectrogram of the Quadrature Doppler signal processed with time frequency distributions (method 4.d). The Quadrature Doppler signal (D+jQ) is a complex signal. Immediately, the positive frequencies of the spectrogram are related to forward flow and the negative frequencies of the spectrogram are related to reverse flow. For this reason, the integral to calculate the instantaneous mean frequency (17) is over the interval (-\(N,\pi\)). The fact that the negative frequencies of the spectrogram are zero was explained in the previous paragraph.
5.b.- Preliminary assessment of a surgical procedure to remove cerebral arteriovenous malformation

Figures 14 to 17 show three boxes. The upper box corresponds to the spectrogram of the signal; the middle box corresponds to the instantaneous mean frequency of the signal (proportional to the instantaneous mean velocity of the flow); and the box below corresponds to the electrocardiogram, which is used as a reference.

As explained in section 2, the flow conditions before surgical intervention are: the presence of a low flow in the feeding artery to the capillaries due to vascular theft by the arteriovenous malformation, and the presence of a high flow in the draining vein from the arteriovenous formation. The first situation is shown in figure 14 (measurement made in point 1 of figure 2) and the second in figure 16 (measurement made in point 3 of figure 2).

On the other hand, the flow conditions after the surgical intervention are: the presence of a restored high flow in the feeding artery of the capillaries due to the gradual extirpation of the arteriovenous malformation, and the presence of a low flow in the draining vein from the arteriovenous formation. The first situation is shown in figure 15 (measurement made in point 1 of figure 2) and the second in figure 17 (measurement made in point 3 of figure 2).

Table 2 shows the average values of the instantaneous mean frequencies (proportional to the instantaneous mean velocity of the flow), of the feeding artery to capillaries and of the draining vein of the arteriovenous malformation, before and during surgery.

<table>
<thead>
<tr>
<th></th>
<th>Before surgery</th>
<th>During surgery</th>
</tr>
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<tbody>
<tr>
<td>Artery of Capillaries</td>
<td>829 [Hz] Low Flow</td>
<td>1683 [Hz] High Flow</td>
</tr>
<tr>
<td>Vein of AVM</td>
<td>1343 [Hz] High Flow</td>
<td>1221 [Hz] Low Flow</td>
</tr>
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Table 2. Average values of the instantaneous mean frequencies (proportional to the instantaneous mean velocity of the flow), of the feeding artery to capillaries and of the draining vein of the arteriovenous malformation, before and during surgery.
6.- Conclusions
This paper shows the application of Doppler flow measurement techniques to assess the development of surgical treatment of cerebral arteriovenous malformations during the intervention.

Also, it is shown the architecture of a Bi-directional Doppler System for Blood Flow Measurement that has already been used to evaluate the bypass quality in coronary revascularization surgeries.

Four suitable spectral analysis methods are described: the use of Short Time Fourier Transform (STFT) or Gabor Transform to analyze the Doppler signal, the use of Time Frequency Distributions to analyze the analytic signal corresponding to the Doppler signal, the use of Flow Separation Algorithms to analyze the Quadrature Doppler signal, and the use of Time Frequency Distributions to analyze the Quadrature Doppler signal.

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