Weighted Graphs to Model Causality

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Abstract – In this paper we explore the role of weighted graphs to evaluate the degree in which a sentence is direct or indirect cause of an effect, to obtain the path with a highest causality degree between two concepts, and on the other hand, given a context, the two concepts with a strongest causal relationship.

Keywords: Weighted causal graphs, causality, causal model, causal representation.

1 Introduction

Causality is an important notion in every field of science. In empirical sciences, causality is a useful way to generate knowledge and provide for explanations. When a quantum physicist calculates the probability of an atom absorbing a photon, he analyses this event as the cause of the atom’s jump to an excited energy level; that is, he tries to establish a cause-effect relationship [1].

Causation is a type of relationship between two entities: cause and effect. The cause provokes an effect, and the effect is a consequence of the cause. Causality can be a direct process when A causes B and B is a direct effect of A, or an indirect one when A causes C through B, and C is an indirect effect of A.

According to Sowa [2], three assumptions described by Born [3] have to be fulfilled to consider a relationship as causal:

1. “Causality postulates that there are laws by which the occurrence of an entity B of certain class depends on the occurrence of an entity A of another class, where the world entity means any physical object, phenomenon, situation, or event. A is called the cause, B the effect”.

2. “Antecedence postulates that the cause must be prior to, or at least simultaneous with the effect”.

3. “Contiguity postulates that cause and effect must be in spatial contact or connected by a chain of intermediate things in contact”.

These three postulates are the base to differentiate causal statements from conditional ones, although causality and conditionality are strongly related.

To represent causation, there have been many approaches, though we will focus in this paper in the representation of causality through causal graphs. The use of causal graphs as a way to represent information has been very present in literature, as Pearl [4] or Spirtes [5], exemplifies. But these representations lack of ponderations in the edges to represent causality degrees. On the other hand, there are studies about causality like the one of Halpern [6] where degrees are assigned to quantify the impact in which a cause provokes an effect, but no causal representation is used. It seems then a novelty proposal to mix the two approaches: use weighted graphs as a way to calculate degrees of direct or indirect causality between two concepts.

2 Weighted Causality Graphs

We define a directed graph $G=(V,E)$, where the vertices of $V$ represent to the concepts selected and $(a,b)\in E$ if the vertex $a$ causes directly the effect of vertex $b$ (without any intermediate vertex).

Following the approach of Chockler, Halpern and Pearl, of assigning causality degrees (on their case, based on responsibility and blame [7] [8]) we will measure the vertex $(a,b)$ with a number in the interval $(0,1]$ indicating the degree on which the vertex $a$ causes directly the effect on vertex $b$, which will be named as $w(a,b)$. The assignment of weights to the edges is done according to the type of causal sentences where $a$ and $b$ appear.

Considering these premises, we can represent indirect causality in graph through causal paths.
Definition 1: we say that the vertex $a$ causes indirectly an effect in vertex $b$ if there is a path with length higher than 1 linking $a$ with $b$ (causal path). The product of the weights of the edges linking $a$ to $b$ will be the degree of that path (being $a$ number in the interval $(0,1]$). The degree of a path with length 1 is the ponderation in the edge in the path.

In some causal graphs there might be cycles, as in the example of Mazlack [9]. We will work with graphs without cycles (recursive graphs), as according to the conditions of causality mentioned before, a cause cannot be the cause and the effect at the same time. This implies that the relationships among nodes in this graph are antisymmetric. If we have the edge $(a,b)$, we cannot have the edge $(b,a)$.

We can quantify indirect causality through a path according to this definition:

Definition 2: the degree that vertex $a$ causes indirectly the effect in vertex $b$, through a path with length greater than 1, is the degree of such path. The degree in with vertex $a$ directly causes the effect in vertex $b$ will be the weight of the edge $(a,b)$ if such edge exists. Otherwise will be 0.

We can quantify total causality, direct or indirect this way:

Definition 3: the total degree in which vertex $a$ causes an effect on vertex $b$, is the addition of all degrees in which vertex $a$ causes the effect on vertex $b$ by all the paths with length greater than 1 from $a$ to $b$ plus the weight of the edge $(a,b)$ in case that exists.

With these definitions we can study the following problems:

Problem 1: given a causal graph, find the maximum of the total degree over all the pairs of vertices and the pairs $a, b$, in which said maximum is attained.

Problem 2: given two vertices $a$ and $b$ of a causal graph, find the path from $a$ to $b$ that will maximize the degree in which vertex $a$ causes directly or indirectly an effect on $b$ through a path, and such maximum degree.

The understanding of both problems in terms of causality is the following. Problem 1 gives us, among the concepts of the graph, those with a stronger cause-effect relationship, in order to include them in a summary of a work modeled by the graph, see for example [10] [11]. Problem 2 gives us the path with highest probability to reach from $a$ to $b$. It can serve to establish an argumental line between nodes $a$ and $b$ in case in a summary we want to link both concepts.

To solve problem 1, we need the following definition similar to the definition of the adjacency matrix in a graph [12]:

Definition 4: given a causal graph with $n$ vertices, $V = \{v_1, ..., v_n\}$, its weighting matrix is $M = (a_{ij})_{n \times n}$, with:

$$a_{ij} = \begin{cases} w(v_i, v_j) & \text{if } (v_i, v_j) \in E \\ 0 & \text{if } (v_i, v_j) \notin E \end{cases}$$

Theorem 1: given a causal graph, with $n$ vertices and a weighting matrix $M$, the maximum element of $M + M^2 + ... + M^{n-1}$, gives the maximum total degree between two vertices of the graph, being the vertices in which the maximum is reached the corresponding to the positions in the matrix of the maximum element.

Proof:

The element $(i,j)$ of $M$ gives the degree in which the vertex $v_i$ causes directly the effect of the vertex $v_j$, and in general, the element $(i,j)$ of $M^k$ gives the addition of the degrees in which vertex $v_i$ causes $v_j$ upon all paths going from $v_i$ to $v_j$ with length $k$. As there are no paths with length greater or equal than $n$ in the graph, the element $(i,j)$ of $M + M^2 + ... + M^{n-1}$ gives the addition of the degrees in which vertex $v_i$ causes the effect in $v_j$ upon all paths going from $v_i$ to $v_j$. So the maximum element of $M + M^2 + ... + M^{n-1}$ is the maximum of the total degrees and the elements in which that maximum is reached correspond to the pair of vertices in which the total degree is maximized.

Another problem that can be solved using causal graphs and that generalizes problem 2 is the following:

Problem 3: given two vertices $a$ and $b$ of a causal graph, arrange the degrees of the paths going from $a$ to $b$. This problem is important because it gives us the three paths by which the vertex $a$ causes in a more relevant way the effect on $b$, which is used in [13] to answer how questions in an automatic form. In such paper they give a ponderation of the causality degrees based on cognitive fuzzy maps, different from the ones used here.

To create an algorithm that solves problems 2 and 3, we have used the following definitions:

Definition 5: given a causal graph with set of vertices:

$$V = \{v_1, ..., v_n\}$$

we define matrix $A = (a_{ij})_{n \times n}$, where:

$$a_{ij} = \begin{cases} w(v_i, v_j)v_j & \text{if } (v_i, v_j) \in E \\ 0 & \text{if } (v_i, v_j) \notin E \end{cases}$$
Definition 6: given a causal graph with \( n \) vertices:

\[ G = (V = \{v_1, ..., v_n\}, E) \]

we define recursively the matrix \( A_k \), \( k = 1, ..., n-1 \) as:

\[ A_k = \left[ a_{ij} \right]_{n \times n}, \quad a_{ij} = \begin{cases} w(v_i, v_j) & \text{if } (v_i, v_j) \in E \\ 0 & \text{otherwise} \end{cases} \]

\[ A_k = A_{k-1} A, \quad k = 2, ..., n-1. \]

The product (symbolic) of matrices is calculated without commute factors.

The algorithm to solve problems 2 and 3 gives the ordering of degrees and obtains the path with highest causal degree between two vertices:

Algorithm 1:

The input will be the causal graph \( G = (V = \{v_1, ..., v_n\}, E) \), the weight on its edges and two indexes \( i, j \in \{1, ..., n\} \)

As output we will have the ordering of the degrees of the paths from \( v_i \) to \( v_j \), the paths corresponding to such order, the maximum degree of the paths from \( v_i \) to \( v_j \) and the path that reaches such maximum degree. To do that, we follow these steps:

- Build matrix \( A \) according to definition 5
- Build matrix \( A_1 \) according to definition 6
- For \( k = 2, ..., n-1 \) find \( A_k = A_{k-1} A \)
- Find \( A_1 + ... + A_{n-1} \) for the matrices \( A_1, ..., A_{n-1} \) obtained in steps 2 and 3.
- Order the terms of the element \((i,j)\) of the matrix obtained in step 4 by its coefficients from greatest to lowest.

The coefficient of the first term of the ordering in step five gives the maximum degree, and the literal part of this term gives the path where this maximum is reached. The ordering of step 5 gives the ordering of the degrees of the paths (taking the coefficients) and the paths corresponding to that ordering (taking the literal part).

Remark: the matrix \( A_1 + ... + A_{n-1} \) describes all the possible paths between each pair of vertices of the graph. So the algorithm obtains all the existing paths linking two vertices of the graph, in a different way as done in [13].

To check our algorithm works, we have used the same causal graph as in [13] (see figure 1). It has 9 vertices, \( v_1, ..., v_9 \), and the following edges: \((v_1, v_2), (v_1, v_4), (v_1, v_9), (v_2, v_3), (v_3, v_4), (v_4, v_5), (v_4, v_6), (v_5, v_8), (v_7, v_9)\).}

![Graph to answer the question How smoking causes death?](image-url)
As there are no paths with length greater than 6 in the graph, we have that $A_k = O$ if $k \geq 7$, being calculated $A_k$ for $k \leq 6$ according to definition 6. We then have:

$$
A_k = \begin{pmatrix}
0 & 0.95 v_2 & 0 & 0.9 v_4 & 0 & 0 & 0 & 0 & 0.85 v_9 \\
0 & 0 & 0.95 v_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.5 v_3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.6 v_4 & 0.6 v_5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.6 v_6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.95 v_7 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6 v_9 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$$
A = \begin{pmatrix}
0 & 0.95 v_2 & 0 & 0.9 v_4 & 0 & 0 & 0 & 0 & 0.85 v_9 \\
0 & 0 & 0.95 v_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.5 v_3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.6 v_4 & 0.6 v_5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.6 v_6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.95 v_7 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6 v_9 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

We order the terms of the element 1, 9 of this last matrix according to the coefficients, what takes us to the resolution of problem 3 for the vertices v1, v9 in this example:

$$
\sum_{i=1}^{s} A_i =
\begin{pmatrix}
0.85v_1v_9, 0.3078v_1v_4v_6v_7v_9, 0.1543275v_1v_2v_3v_4v_6v_7v_9, 0.1944v_1v_4v_5v_8v_9, 0.09747v_1v_2v_3v_4v_5v_8v_9
\end{pmatrix}
$$

So, the path with highest causal degree is v1v9 (direct edge), with ponderation 0.85, what solves problem 2 in this case.

Remark: according to the obtained ordering, the three paths with highest causal degree are v1v9, v1v4v6v7v9, and v1v4v5v8v9, with degrees 0.85, 0.3078, and 0.1543275 respectively. These are the same paths as in [13] but with this method we deep higher: In [13], both paths v1v4v6v7v9, and v1v4v5v8v9 had the same causal degree, while with this method, the first one has a higher causal degree. Another remarkable fact is that the discarded paths v1v2v3v4v6v7v9 and v1v2v3v4v5v8v9 both had the same causal degree in [13] (eventually), having a different degree with our method.

3 Conclusions

One of the main problems of causal graphs is to establish which path is the best one linking two nodes. In this paper we have presented a contribution that establishes a criterion to select the “best” causal path between two nodes according to the weight in the edges of such path. This ponderation may serve us in the future for three main purposes. The first one is to select the most important nodes according to its causal weight when creating a summary. The second, would be when asking a question, select the causal path that links two nodes with the highest degree of causality to include those nodes in the answer of the question. The third use would be to remove redundant nodes in a causal graph. For instance in the graph included in [13] we had “tobacco use” and “smoking”. With this measurement we are able to select the node with a highest ponderation to work with.

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5 References


