Adaptive Mesh Refinement, Interface Reconstruction and Numerical Algorithms for Plasma 3-T Radiation Diffusion

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Abstract—A set of numerical procedures is proposed to resolve material interface in numerical simulations for plasma 3-T radiation diffusion. For that we first introduce interface reconstruction on the top of cell-based adaptive mesh refinement (AMR). A numerical cell of the resulting mesh through interface reconstruction is a general polygon or polyhedron. Furthermore, we proposed a numerical approach for the set of nonlinear equations. The numerical approach not only is second order accurate in both space and time, but also gives correct steady states when the size of time step is very large. The discontinuity of material properties between different materials is correctly treated based on the governing physics principle for general polyhedral meshes and full nonlinearity. The treatment is exact for arbitrarily strong discontinuity. The approach is fully nonlinear for the full nonlinearity in plasma 3-T radiation diffusion equations. Three temperatures are fully coupled and are updated simultaneously. The scheme is general in two and three dimensions on polyhedral mesh. The features of approach will be demonstrated through numerical examples.

Keywords: AMR; interface; diffusion; coupling; implicit

1. Introduction

Typical Eulerian methods have difficulty in keeping sharp material interfaces, leaving questions as to what is correct physical behavior or artifact of numerical methods. This sometimes limits Eulerian methods usefulness for certain types of calculations. With adaptive mesh refinement (AMR), and treatment of material interfaces, the range of calculations for which the Eulerian approach can be applied has the potential to be greatly expanded.

In this paper, we will propose an approach to better resolve material interface through AMR, interface reconstruction, best numerical treatment for the discontinuity of material property, and better numerical methods for plasma 3-T radiation diffusion equations. Any aspect of the approach is significant for numerical simulations in a multi-physics code. This paper shows the power of some of meshing techniques, such as AMR, interface reconstruction, better algorithms, in numerical simulations of physics systems. We take an approach of cell-based AMR for better refinement efficiency. For interface reconstruction, we include the case of many materials in one numerical cell, which are often encountered in numerical simulations of Eulerian codes.

For problems with multi-materials of dramatically different properties, a correct treatment for the discontinuity of material properties is important. One approach for the calculation of flux near material interfaces is to use mathematical approximations, for example, some weighted average of two adjacent materials. This approach would introduce numerical errors when thermal properties of two materials are very different. In this paper, we will give a general formulation to calculate the effective diffusion coefficients of radiation, electrons, and ions at an interface between two arbitrarily different materials within the framework of polyhedral meshes. Particularly, the effective diffusion coefficient for radiation is fully nonlinear.

There are many investigations on numerical methods for 3-T radiation diffusion equations. Some work on unstructured meshes but are only first order accurate in time. Some are second order accurate but do not work for large time steps. Some work well for clean cells (i.e., cells with single material), but do not deal with mixed cells. Some are designed for mixed material, but lack special consideration for the discontinuity at material interfaces. Some work for single diffusion equation, but don’t work well for the coupling between radiation and material. Some deal with linearized 3-T radiation diffusion equations, but do not pay enough attention on the nonlinearity. In this paper, we will present a numerical method for two- and three-dimensional plasma 3-T radiation diffusion equations for systems of multi-materials with arbitrarily different thermal properties.

2. Cell-based Adaptive Mesh Refinement

In this section, we will describe the cell-based adaptive mesh refinement implemented in a multi-physics code. The principal purpose of the refinement is to resolve material interfaces and shock fronts.

Assume there is a circular shock front or material interface that passes through the red cells shown in the left image in Fig.1. Therefore these red cells are refined as shown in the middle image in Fig.1. If it is allowed to further refine the cell, we will further refine the red cells in the middle image because these cells are still mixed cells. At this point, we have to note that the cells marked with "x" in the middle image are neighboring with the red cells horizontally, or vertically, or diagonally. When we refine the red cells, we also refine the cells marked with "x", so that the resulting
mesh is "smoothly" refined, i.e., any adjacent cell is at the cell level, or one level up, or one level down, as shown in the image at the right of Fig. 1. Let us call this process to refine the cells marked with "x", cascading. We have to point out that when any red cell in the middle image belongs to a different processor from its neighboring cell marked with "x", we have to have one communication to guarantee that the cell marked with "x" is also refined.

After every time step during a simulation, four cells in two dimensions or eight cells in three dimensions could be coarsened into one cell if a shock front or material interface is moving out of the cells and any of these four or eight cells are not marked to be refine through the cascading process described above.

In our cell-based AMR, cells are listed by their cell ids, nz with its center and width are specified. For most physics calculations, the connectivity between cells must be built before a calculation could proceed. There are several ways to establish the connectivity. Some take advantage of the nature of structured cells, but at the cost of memory footprint. Others describe the connectivity in the way for unstructured meshes. We take advantage of the nature of structured cells but with the minimum memory cost. Two arrays, nlow(nz, idim) and nhgh(nz, idim), are used for the purpose. Here idim is the index for dimension. The value nlow(nz, idim) gives the cell id of the lower side of the dimension idim. In the case of one-dimensional problem, this neighboring cell is uniquely determined. In the multi-dimensional case, nlow(nz, k) further refers to the cell that is at the lower end of any dimension other than the kth one. Similarly, nhgh(nz, k) gives the neighboring cell nz at the high end of the kth dimension, but at the lower end of any dimension other than the kth one. Figure 2 illustrates the definitions of array nlow and nhgh for the cell nz. It should be mentioned that nlow and nhgh give only the local ids of cells.

The reason we could effectively use the concept of nlow and nhgh is that mesh is gradually refined, i.e., an immediate neighboring cell of any cell can only be one level higher or lower. This assumption is also guaranteed across processor interfaces.

There are many methods to generate the connectivity arrays. We use a local (i.e. my own processor) KD-tree to set up nlow and nhgh. Each identity in the KD-tree is a rectangular cell. After the array nlow and nhgh are setup, the tree is not needed anymore until the mesh is changed, typically after one time step.

3. Interface Reconstruction for Material Interfaces

Fig. 3: Illustration of the normal direction of reconstructed material interface in two (left) and three (right) dimensions.

To resolve the sub-cell structure within a mixed cell, we reconstruct interfaces between materials within each mixed cell from the distribution of volume fraction of the material. A typical interface reconstruction in numerical simulations
is through a single line in two-dimension or a single plane in three dimensions within a mixed cell with two materials.

Our procedure to reconstruct interfaces between materials follows. We first consider a situation in which a mixed cell contains only two materials. For a given mesh and volume fraction \( f_m \) of one material in each cell, we first approximately obtain the gradient of the volume fraction, \( \nabla f_m \), on each cell. The direction of the gradient \( \nabla f_m \) is shown in Fig. 3 in two and three dimensions. After obtaining the direction of the gradient within the cell, we draw a line (in the two dimensional case) or plane (in the three-dimensional case) within the cell perpendicular to the gradient, and then we move the line or plane along the direction of gradient until the two volumes separated by the line or plane within the cell match the volumes of the two materials in the cell. The interfaces are constructed cell by cell. Figure 4 displays sub-cells and reconstructed interfaces in a three-dimensional mesh. In Fig. 5 we show only mixed cells, and each color in the image represents one material. We will solve diffusion equations on the resulting reconstructed mesh, which typically includes arbitrary polygons or polyhedrons.

![Image](image.png)

Fig. 5: The reconstructed material interfaces of four materials (left image) and ten materials in three dimensions.

4. Plasma 3-T Radiation Diffusion Equations

In a system of plasma with radiation, for many problems, the temperatures of radiation, electrons, and ions, \( T_r \), \( T_e \), and \( T_p \), are not necessarily in equilibrium. The set of 3-T radiation diffusion equations is

\[
\begin{align*}
\frac{\partial \phi}{\partial t} &= -\nabla \cdot \mathbf{F}_r + S_r, \\
C_{ve} \frac{\partial T_e}{\partial t} &= -\nabla \cdot \mathbf{F}_e - S_r + S_e, \\
C_{vp} \frac{\partial T_p}{\partial t} &= -\nabla \cdot \mathbf{F}_p - S_e.
\end{align*}
\]

Here \( \phi \equiv T^4_r \), \( a \) is the radiation constant, \( C_{ve} \) and \( C_{vp} \) are heat capacities of electrons and ions, and they are related with material mass density \( \rho \) and specific heat capacities \( c_{ve} \) and \( c_{vp} \) through \( C_{ve} \equiv \rho c_{ve} \) and \( C_{vp} \equiv \rho c_{vp} \). \( \mathbf{F}_r, \mathbf{F}_e, \) and \( \mathbf{F}_p \) are energy fluxes of radiation, electrons, and ions respectively, and they are defined as

\[
\begin{align*}
\mathbf{F}_r &= -\sigma_r \nabla \phi, \\
\mathbf{F}_e &= -\sigma_e \nabla T_e, \\
\mathbf{F}_p &= -\sigma_p \nabla T_p.
\end{align*}
\]

Here \( \sigma_r, \sigma_e, \) and \( \sigma_p \) are diffusion coefficients of radiation energy and temperatures of electrons and ions. \( \sigma_p \) is inversely proportional to Rosseland opacity that includes atomic processes such as Bremsstrahlung, photo-ionization, line-absorption, and Thompson scattering. The source terms in Eqs.(1) are defined as

\[
\begin{align*}
S_r &= a e p c_p (T^4_e - T^4_r), \\
S_e &= C_{ve} \kappa_p (T_p - T_e).
\end{align*}
\]

In Eqs.(3), \( c \) is the light speed, \( \kappa_p \) Planck mean opacity, and \( \kappa_{pe} \) the coefficient for interaction between electrons and ions. The term, \( S_r \), in the first two equations in Eqs.(1) allows for the exchange of energy between radiation and electrons.

5. Numerical Schemes

Let us illustrate our numerical scheme through a simple diffusion equation for temperature \( T(t, \mathbf{r}) \)

\[
\frac{\partial u(T)}{\partial t} + \nabla \cdot \mathbf{F}(T) = S(T).
\]

Here \( u(T) \) is energy density, a function of temperature, and \( \mathbf{F} \) is energy (or heat) flux, defined as

\[
\mathbf{F} = -\kappa \nabla T.
\]

The coefficient \( \kappa \) represents diffusion coefficient, and it may be very different for different materials. \( S \) is a source term.

In general, Eq.(4) is a nonlinear equations, and the function \( u(T) \), the coefficients, \( \kappa, \kappa_0, C_v, \) and \( \rho \) may be dramatically different for different materials involved in one simulation. The nonlinearity and discontinuity of material properties are two difficulties in numerical simulations.

Since resulting meshes through reconstruction of interfaces between materials are unstructured that include arbitrary polygons and polyhedrons, we will discuss numerical schemes for Eq.(1) or Eq.(4) on a general polyhedral mesh in two and three dimensions. After we develop our numerical scheme on unstructured meshes, we will simplify apply the scheme to polyhedral meshes generated from reconstruction of material interface.

5.1 Second-order Accuracy in Time

In the Fig.6, we illustrate a cell of an unstructured mesh, the red cell, which thermally interacts with its neighboring cells marked with the green color through thermal flux across the interface between the red cell and each green cell. For example, the \( i^{th} \) cell interacts with \( k^{th} \) cell through the interface \( A_{ik} \). Considering the red cell as shown in Fig.6, we integrate Eq.(4) over the cell and one time step \( 0 < t < \Delta t \)
with $\Delta t$ the size of time step. After converting the volume integral into a surface integral over the enclosed surface of the cell for the term of flux, we get the following equation,

$$u^n_i = u_i - \frac{\Delta t}{\Delta V_i} \sum_{k \in N_i} F_{ik} A_{ik} + S_i \Delta t. \tag{6}$$

Here $\Delta V_i$ is the volume of the cell, $N_i$ is a set of cells, each of which neighbors the cell $i$ with a common non-vanishing interface as shown by the green cells in Fig.6. $A_{ik}$ is the area of the interface between cell $i$ and cell $k$. $S_i$ is the source term defined as

$$S_i = \frac{1}{\Delta t \Delta V_i} \int_0^{\Delta t} \int_{\Delta V_i} S(t, r) dV dt.$$ 

The superscript $n$ in Eq.(6) stands for the new time $\Delta t$. $u^n_i$ is the cell averaged $u$ at $t = \Delta t$ and it is defined as

$$u^n_i = \frac{1}{\Delta V_i} \int_{\Delta V_i} u(\Delta t, t) dV.$$ \tag{7}

$u_i$ is similarly defined at $t = 0$. $F_{ik}$ is the time-averaged flux across the common interface $A_{ik}$ and is defined as

$$F_{ik} = \frac{1}{A_{ik}} \int_0^{\Delta t} \int_{A_{ik}} F(t) \cdot dA dt. \tag{8}$$

The summation in Eq.(6) is over the set $N_i$, the neighboring cells of cell $i$. We would like to point out that Eq.(6) is exact, since we have not made any approximation yet.

If we know how to calculate the time-averaged flux needed in Eq.(6), we then obtain $u$ at $t = \Delta t$ from the given initial condition and source function. Therefore, one of the major tasks in numerical methods is to approximately find the time-averaged flux.

If the time-averaged flux is replaced by its initial value, the approximation results in Euler forward method. If the flux is replaced by the value at the end of the time step, the scheme is called Euler backward scheme. Both Euler forward and backward schemes are first order accurate in time. One of the good features of Euler backward scheme is that numerical errors undergo quick damping for large time steps, this feature is very useful for steady states, and it is important for a time-dependent problem if materials in the problem have very different properties. If the time-average flux in Eq.(6) is replaced by the averaged of fluxes at $t = 0$ and $t = \Delta t$, the result is the Crank-Nicolson scheme, which is second order accurate in time. But the numerical errors in Crank-Nicolson scheme do not damp out for large time steps.

To introduce quick damping for numerical errors within the second order accuracy, we introduce an additional time level, as demonstrated in the scheme for hydrodynamics [1,2], for radiation hydrodynamics [8], and for heat conduction on structured mesh [3]. Within the second order accuracy, we approximately evaluate the time-averaged flux at $t = \Delta t/2$. Thus Eq.(6) becomes

$$u^n_i = u_i - \frac{\Delta t}{\Delta V_i} \sum_{k \in N_i} F_{ik}^h A_{ik} + S_i(T_i^+) \Delta t. \tag{9}$$

Here the superscript $h$ stands for the evaluation at the half time step, $t = \Delta t/2$. $F_{ik}^h$ is the flux evaluated at the half time step and at the interface between cells $i$ and $k$, defined as

$$F_{ik}^h = \frac{1}{A_{ik}} \int_{A_{ik}} F(\Delta t/2, t) \cdot dA.$$ \tag{10}

There are a few approaches that can be used to approximately calculate $F_{ik}^h$. For example, $F_{ik}^h$ could be approximately calculated through $(T_i + T_k^h)/2$ or $(F_{ik} + F_{ik}^h)/2$, here again the superscript $n$ stands for the evaluation at the new time $t = \Delta t$. But the resulting scheme through this will not work for large time steps, although it is second order accurate.

Our approach is to introduce a new set of unknowns at $t = \Delta t/2$, $u^h_i$ or $T^h_i$, which are defined as

$$u^h_i = \frac{1}{\Delta V_i} \int_{\Delta V_i} u(\Delta t/2, r) dV.$$ \tag{11}

Since $u^h_i$ or $T^h_i$ is a new set of unknowns, like $u^n_i$ or $T^n_i$, we need an equation to determine $u^h_i$ or $T^h_i$. For this purpose, we integrate Eq.(4) on the volume $\Delta V_i$ and the first half time step $0 \leq t \leq \Delta t/2$. After converting the volume integral of the flux into surface integral, we obtain the following.

$$u^h_i = u_i - \frac{\Delta t}{2 \Delta V_i} \sum_{k \in N_i} F_{ik}^h A_{ik} + \frac{1}{2} S^h_i \Delta t. \tag{12}$$

Here $F_{ik}^h$ and $S^h_i$ are defined as

$$F_{ik}^h = \frac{2}{\Delta t A_{ik}} \int_{A_{ik}} F(t) \cdot dA dt.$$ \tag{13}

$$S^h_i = \frac{2}{\Delta t A_{i}} \int_{A_{i}} S(t, r) dV dt.$$
must not, even partially, depend on the initial temperature if we want the property of the scheme for large time steps. Therefore, instead of interpolation through \( F_{ik} \) (the initial flux) and \( F_{ik}^n \), we use extrapolation through \( F_{ik}^h \) and \( F_{ik}^n \) to find \( F_{ik}^h \). Therefore the linear form of \( F_{ik}(t) \) with time is uniquely determine by its values at \( t = \triangle t/2 \) and \( t = \triangle t \), \( F_{ik}^h \) and \( F_{ik}^n \). Thus, the time-averaged flux and sources defined in Eq.(12) is approximately obtained through the following approximation:

\[
F_{ik}^h \approx \frac{3}{2} F_{ik}^n - \frac{1}{2} F_{ik}^h,
\]

Here \( F_{ik}^n \) is the flux calculated from temperature \( T_i^n \) at \( t = \triangle t/2, F_{ik}^h \) is the flux calculated from temperature \( T_i^h \) at \( t = \triangle t/2, S_i^n \) and \( S_i^h \) are source term \( S(T) \) evaluated at \( T_i^n \) and \( T_i^h \), and \( S_i^n \) is a function of \( T_i^n \), and \( S_i^h \) is a function of both \( T_i^n \) and \( T_i^h \). Except for the calculation of flux from temperature, we have developed our difference equations as shown in Eqs.(9,12) with Eqs.(14,15) for the diffusion equation (4).

5.2 Effective Diffusion Coefficients across Material Interface

Now we move to the calculation of the flux from temperature needed in Eqs.(9,12). As we stated before, there are possibly discontinuities in diffusion coefficient between cells, and thus spatial derivatives is discontinuous across the interfaces although temperature is continuous. Therefore, we can not use Taylor expansion across interface to approximately evaluate the flux if there is a strong discontinuity across an interface.

Our calculation for the flux is based on the conservation law of energy. From the fact that the energy flux getting into an interface between two materials is equal to the flux getting out of the interface, we get the flux across the interface

\[
F_{ik} = -\hat{k}_{ik}(T_K - T_i),
\]

with definition

\[
\hat{k}_{ik} = \frac{\kappa_i \kappa_k}{l_k \alpha_i \kappa_i + l_i \alpha_k \kappa_k} \alpha_i \alpha_k.
\]

As shown in Fig.7, here \( l_i \) (or \( l_k \)) is the distance vector from the center of cell \( i \) (or \( k \)) to the center of the interface, \( l_i \) and \( l_k \) are their magnitudes, and \( l_i \alpha_i \) and \( l_k \alpha_k \) are the magnitudes of projection of \( l_i \) and \( l_k \) along the normal direction of the interface. We would like to point out that the diffusion coefficients \( \kappa_i \) and \( \kappa_k \) are values defined on cells. Typically, they are calculated from analytical formulae or obtained from lookup tables for given values of material density and temperature on the cells.

5.3 Scheme for Plasma 3-T Radiation Diffusin

For plasma 3-T radiation diffusion equations, Eq.(1), we could get the following set of nonlinear difference equations,

\[
[a_{i}T_{i} + \Delta t \sum_{k \in N_{i}} (\sigma_{sk} A_{ik})]T_{i} - \tau_{i}T_{i} = \gamma_{i} [T_{i} + T_{k} - 2T_{i}],
\]

\[
= a_{i}T_{i} + \Delta t \sum_{k \in N_{i}} (\sigma_{sk} A_{ik} T_{ik}),
\]

\[
- \tau_{i}T_{i} + [C e_{i} \alpha_{i} \kappa_{i} \Delta t]T_{i} = C e_{i} T_{i} + \Delta t \sum_{k \in N_{i}} (\sigma_{sk} A_{ik} T_{ik}),
\]

\[
= C e_{i} T_{i} + \Delta t \sum_{k \in N_{i}} (\sigma_{sk} A_{ik} T_{ik}).
\]

In Eqs.(18,19), \( \gamma_{i} \) and \( \tau_{i} \) are defined as

\[
\gamma_{i} = \frac{[T_{i}^2] + T_{i}^2}{[T_{i}^2 + T_{k}^2]}(T_{i} + T_{k}),
\]

\[
\tau_{i} = \frac{\kappa_{k} e_{i} [T_{i}^2 + T_{k}^2]}{[T_{i}^2 + T_{k}^2]}(T_{i} + T_{k}) \Delta t,
\]

The coefficients \( \sigma_{sk}, \sigma_{sk} \) and \( \sigma_{sk} \) in Eqs.(18-20) are

\[
\sigma_{sk} = \frac{\alpha_{i} \alpha_{k} \sigma_{sk} \sigma_{ek}}{l_k \alpha_i \sigma_{sk} + l_i \alpha_k \sigma_{ek}}.
\]
\[ \sigma_{rik} = \frac{\alpha_i \alpha_k \sigma_{rki} \sigma_{pk}}{t_k \alpha_i \sigma_{pi} + t_i \alpha_k \sigma_{pk}}, \]  
\[ \sigma_{rki} = \frac{\alpha_i \alpha_k \sigma_{rki} \sigma_{rki}}{t_k \alpha_i \sigma_{ri} + t_i \alpha_k \sigma_{rki}}(T_{rk}^2 + T_{ri}^2)(T_{rk} + T_{ri}). \]  

\( \sigma_{rki} \) is the value \( \sigma_{rki} \) evaluated at \( T_{ri} \) and \( T_{rk} \). The system of Eqs.(18-20) is nonlinear because \( \sigma_{rki} \), \( \gamma_{ri} \), and \( \tau_{ri} \) depend on \( T_{ri} \), \( T_{pi} \) and \( T_{rk} \).

The set of nonlinear difference equations, Eqs.(18-20), is a simplified case of the set of coupled equations involving the half-time step [15]. Equations (18-20) could be further linearized, to get the following set of equations,

\[ [\alpha \gamma_i + \frac{\Delta t}{\Delta V_i} \sum_{k \in N_i} (\bar{\sigma}_{rik} A_{ik}) + \tau_i] T_{ri}^m - \tau_i T_{ei}^m = \]

\[ \alpha \gamma_i T_{ri} + \frac{\Delta t}{\Delta V_i} \sum_{k \in N_i} (\bar{\sigma}_{rik} A_{ik} T_{rk}^m), \]

\[ - \tau_i T_{ri}^m \]

\[ + [C_{vei} + \frac{\Delta t}{\Delta V_i} \sum_{k \in N_i} (\bar{\sigma}_{vei} A_{ik}) + \tau_i + C_{vei} \kappa_{pei} \Delta t] T_{ei}^m \]

\[ - C_{vei} \kappa_{pei} \Delta t T_{pi}^m = C_{vei} T_{ei} + \frac{\Delta t}{\Delta V_i} \sum_{k \in N_i} (\bar{\sigma}_{vei} A_{ik} T_{pi}^m), \]

\[ - C_{vei} \kappa_{pei} \Delta t T_{ei}^m \]

\[ + [C_{pi} + \frac{\Delta t}{\Delta V_i} \sum_{k \in N_i} (\bar{\sigma}_{pi} A_{ik}) + C_{pi} \kappa_{pei} \Delta t] T_{pi}^m \]

\[ = C_{pi} T_{pi} + \frac{\Delta t}{\Delta V_i} \sum_{k \in N_i} (\bar{\sigma}_{pi} A_{ik} T_{pi}^m). \]

Here \( \sigma_{rik} \), \( \gamma_{ri} \), abd \( \tau_{ri} \) are \( \sigma_{rki} \), \( \gamma_{rki} \), abd \( \tau_{rki} \) but evaluated at the temperatures at \( t = 0 \), \( T_{ei} \), \( T_{pi} \) and \( T_{ri} \).

### 6. Numerical Examples

In this section we will provide numerical examples to show the features of the numerical approach. The first example is for the accuracy of numerical algorithm for a simple Eq.(4). The left plot in Figure 8 shows the error at \( t = 0.1 \) from different simulations with different space resolutions for fixed time step \( \Delta t = 10^{-8} \). The slope of the line segment from two points at \( \Delta x = 5.0 \times 10^{-4} \) and \( \Delta x = 10^{-3} \) is about 4.5, and the slope of the other line segment in the figure is about 2.0. Therefore, the scheme for the simple problem is second order accurate in space for this problem within these spatial resolutions. To show the order of accuracy in time, we carried out another set of simulations with different sizes of time step but a fixed space resolution. The plot at the right of Figure 8 shows the error of numerical solutions with different time resolutions. From least square fitting, we could find the slope of the fitting line, which is about 1.9. Therefore the scheme is close to the second order accuracy in time for this problem under these spatial and temporal resolutions.

The second example is for steady states with a large time step for plasma 3-T radiation diffusion equations. There are two materials in the simulation domain, and the material interface is at \( r = 0.6 \). The diffusion coefficients, \( \sigma_r \), and \( \sigma_p \), are \( 1.009 \times 10^6 \), \( 3.690 \times 10^2 \), and \( 1.160 \times 10^{-2} \) in the inner region, and are \( 1.697 \times 10^3 \), \( 7.372 \), and \( 5.632 \times 10^{-8} \) in the outer region. The electron and ion heat capacities, \( C_{ve} \) and \( C_{vp} \), are \( 5.293 \times 10^3 \) and \( 1.502 \times 10^3 \) in the inner region, and \( 5.882 \times 10^3 \) and 91.70 in the outer region. The coupling coefficients \( \kappa_{ve} \) and \( \kappa_{pe} \) are \( 2.912 \times 10^5 \) and \( 8.148 \times 10^4 \) in the inner region, and \( 2.424 \times 10^4 \) and \( 1.907 \times 10^5 \) in the outer region. Initial temperatures of radiation, electrons, and ions are 20 in the inner region, and are \( 882 \), \( 372 \), and \( 372 \) in the outer region. Fixed temperatures 20 and 1 are imposed on the boundaries of the simulation domain. Figure 9 shows the solutions of difference Eqs.(18-20) for temperatures of radiation, electrons, and ions after one large time step \( \Delta t = 10^{10} \), i.e., the steady state of the problem after one time step. Actually, the three temperatures are the same as they should be for the steady state. We show the actual mesh in the middle image of the figure. Figure 10 is the steady state after one time step for the same problem in three dimensions.
Fig. 11, we show the solutions after one time step obtained either in the inner or outer region, and they are 1.0 and 4.0. In temperatures of radiation, electron, and ion are the same in and between electrons and ions. Initially, the large heat capacity, and strong coupling between radiation and electrons and between electrons and ions. The image on the left is the radiation temperature, the middle one is the temperature of electrons, and the one on the right is the temperature of ions.

Fig. 10: The steady state of a three-dimensional problem obtained through Eqs.(18-20) after one time step $\Delta t = 10^{10}$. Without approximations. As a result, the scheme is fully nonlinear, second-order accurate in both space and time, valid for a large time steps. In the scheme, the temperatures of radiation, electrons, and ions are fully coupled and are updated simultaneously. We have demonstrated the properties of the scheme through numerical examples, including steady states and time-dependent problems.

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