Performance Comparison of Type-1, Interval Type-2 and Generalized Type-2 Fuzzy Logic Controllers

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Abstract— Nowadays Fuzzy logic in control applications is a well-recognized alternative, and this is thanks to its inherent advantages, such as its robustness. However, the Type-2 Fuzzy Logic approach, allows better management of uncertainty in the model. Type-2 Fuzzy Logic has shown significant improvement in image processing applications; however it is also important to analyze its impact in a controller performance.

This paper is presenting a comparison in the performance of Type-1, Interval Type-2 and Generalized Type-2 Fuzzy Logic Controllers, in order to generate criteria to decide which controller is better in specific applications.

The plants considered in the performed experimentation are two benchmark plants and we report the Integral Squared Error (ISE), Integral Absolute Error (IAE) and Integral Time-weighted Absolute Error (ITAE) performance metrics, and also another important metric reported is the execution time.

Based on the experimental results, Fuzzy Controllers selection criteria are proposed according the performance and execution time demands.

Keywords— Fuzzy Control, Generalized Type-2 Fuzzy Logic, Interval Type-2 Fuzzy Logic.

I. INTRODUCTION

Control systems is one of the most well recognized fuzzy logic applications [1]–[6], providing robustness [7]–[11], free-model design [12], and better performance than classic alternatives, and this is mainly thanks to their non-linear modeling abilities. For this reason, Fuzzy Logic Controllers (FLC) are very useful in complex plants [13]–[15].

However, with the advances in fuzzy logic, and the emergence of type-2 fuzzy logic theory, and their implications expanding the options of FLC [16], allowing to design IT2 FLCs and GT2 FLCs, but now it is important to find out what are the impact of these advances, and justify the use of more complex controllers. The present paper aims at analyzing the performance of T1 FLC, IT2 FLC and GT2 FLC to find the advantages and disadvantages of these types of controllers against the others.

The reminder of the paper is organized as follows. Section II presents some basic concepts of fuzzy logic, Section III reports the experimental results with two benchmark plants and finally Section IV offers the conclusions.

II. FUZZY LOGIC BACKGROUND

A. Type-1 Fuzzy Logic

In the literature [17], we can find the definition of a Fuzzy Set (FS) described by the membership function (MF), this first definition is now known as Type-1 Fuzzy Sets, and is expressed with the following Eq. (1).

$$ A = \{(x, \mu_A(x)) | x \in X\} $$ (1)

Where $\mu_A(x): X \rightarrow [0,1]$ and defines the membership degree of $x$ to the set $A$.

Fig. 1 shows the structure of a Mamdani Fuzzy Inference System (FIS). A fuzzy system [18] is composed of the fuzzifier, rules, inference and defuzzifier.

![Mamdani T1 FIS](image)

Fig. 1 Mamdani T1 FIS

B. Interval Type-2 Fuzzy Logic

Based on the original concepts of Fuzzy Sets [19], Interval Type-2 Fuzzy Sets (IT2 FS) are defined as a mathematical approach of fuzziness for considering that the uncertainty in the model can be directly defined. Several advances in this approach have been realized in recent years. The mathematical expression of an IT2FS is as follows (Eq. 2).

$$ \tilde{A} = \{((x, u), 1) | \forall x \in X, \forall u \in J_u \subseteq [0,1]\} $$ (2)

Where $X$ is the primary domain that represents the membership degree of the fuzzy set and $J_u$ is the secondary domain related with the uncertainty, which in this case is always equal to 1.
IT2 MF can be used based on two T1 MFs, and these are known as the upper MF and lower MF and the Footprint of Uncertainty (FOU) [20] which is between both of them, and Fig. 2 illustrates this representation.

On other hand, the structure of a FIS based on IT2 FL is illustrated in Fig. 3.

Where it can be observed that the outputs are computed as two T1 FIS with a new block of a Type-Reduction process [21]. This process has been widely studied by several researchers in order to reduce the computational cost and there exist many variations of type-reduction. The most used and the one that inspired all variations is the one that was proposed by Karnik and Mendel in [22] and is the KM Type-Reduction. The continuous mode of KM [23] is presented in Eq. 3 and consists in obtaining the critical points of the centroid of the combinations of the upper and lower equations.

\[
\begin{align*}
\min C_L &= \frac{\int_{a}^{b} \mu_U(s)ds \int_{a}^{b} \mu_L(s)ds}{\int_{a}^{b} \mu_U(s)ds + \int_{a}^{b} \mu_L(s)ds} \\
\max C_R &= \frac{\int_{a}^{b} \mu_U(s)ds + \int_{a}^{b} \mu_L(s)ds}{\int_{a}^{b} \mu_U(s)ds + \int_{a}^{b} \mu_L(s)ds}
\end{align*}
\] (3)

A graphical illustration of the type-reduction process is presented in Fig. 4.

A. Generalized Type-2 Fuzzy Logic

The mathematical expression of Generalized Type-2 Fuzzy Sets is similar to the IT2 FS expression, and it is because, IT2 FSs, are special cases of GT2 FSs (Eq. 4).

\[
\tilde{A} = \{(x, u) | x \in X, \forall u \in J_u \subseteq [0,1]\}
\] (4)

This theoretical expression can be represented by the union of special cases of IT2 FS known as \( \alpha \)-planes [24] (Eq. 6), and each \( \alpha \)-plane is equivalent to an IT2 FS (Eq. 5).

\[
\tilde{A}_\alpha = \{(x, u) | x \in X, \forall u \in J_u \subseteq [0,1]\}
\] (5)

\[
\tilde{A} = \bigcup \tilde{A}_\alpha
\] (6)

Based on this, [25] proposes the system output as follows (Eq. 7).

\[
\tilde{y} = \frac{\sum \alpha \tilde{a}_\alpha}{\sum \alpha}
\] (7)

A representation of GT2 FIS, based on \( \alpha \)-planes, is illustrated in Fig. 6.
B. Type-2 MFs representations

In the present work, the representation of the T2 MF is designed based on a T1 MF in order to compare the performance of the different controllers using the same structure for the fuzzy sets.

We consider the trapezoidal membership function [17] as expressed in Eq. 8.

\[
T1MF = \text{trapmf}(x, [a, b, c, d])
\]  

(8)

Based on this T1 MF we propose the design of an IT2 MF as follows (Eq. 9).

\[
IT2MF = \frac{\text{trapmf}_{\text{upper}}(x, [a - \frac{u}{2}, b, c, d + \frac{u}{2}])}{\text{trapmf}_{\text{lower}}(x, [a + \frac{u}{2}, b, c, d - \frac{u}{2}])}
\]  

(9)

Where \(u\) is a variable proposed to change the FOU of the IT2MF, \(a, b, c, d\) are the parameters of the T1 trapezoidal membership functions, and the design considers the upper and lower membership functions. The graphical representation of Eq. 9 is illustrated in Fig. 7.

![Fig. 7 IT2 TrapMF](image)

As can be observed, this approach proposes to consider only symmetric MFs, in this study.

III. EXPERIMENTS

The experiments consist, first in comparing the computation time of the FLCs, and then, compare the control surfaces with respect to the T1 FLC, in order to find the impact of the FOU in the controller. After that, finding the performance of FLCs with and without noise by variation of the FOU by changing \(u\) of Eq. 9, and finally obtain the average of 30 experiments, reporting the SSE, ISE, IAE and ITAE performance metrics [26].

C. Computation time.

The first metric to compare the controllers is the computation time of the particular controller, and the relevance of this metric depends on the environment of the application.

Table I reports the results of the simulation time; these results are the average of 30 experiments with 10000 fuzzy evaluations each experiment.

<table>
<thead>
<tr>
<th></th>
<th>T1</th>
<th>IT2</th>
<th>GT2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>0.6156</td>
<td>1.9075</td>
<td>1.5725</td>
</tr>
</tbody>
</table>

The result of GT2 is an \(\alpha\) linear function with a slope of 1.0725, and Fig. 8 shows graphically these results.

![Fig. 8 Computation Time vs Number of Alpha-planes (GT2, IT2 and > T1)](image)

We can observe how much complex are the GT2 FLCs compared with respect to T1 and IT2 FLCs.

D. Water level control plant

The water level control plant [1] (Fig. 9) is commonly used to compare the performance of Fuzzy Logic Controllers (FLC), and the problem consists in achieving level control of a tank based on the control of the valve.
The FLC has two inputs (Level and Rate) and one output (Valve). Table II summarizes the rules of the fuzzy controller.

**TABLE II KNOWLEDGE BASE**

<table>
<thead>
<tr>
<th>Level</th>
<th>Rate</th>
<th>Valve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ok</td>
<td>No Change</td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>Open Fast</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>Close Fast</td>
<td></td>
</tr>
<tr>
<td>Okay</td>
<td>Positive</td>
<td>Close Slow</td>
</tr>
<tr>
<td>Okay</td>
<td>Negative</td>
<td>Open Slow</td>
</tr>
</tbody>
</table>

**CONTROL SURFACES**

Based on Table II, Figs. 10, 11 and 12 show the control surfaces of the FLCs based on T1, and their absolute difference with respect to IT2 and GT2, respectively.

**Fig. 10 T1 Control surface**

Fig. 10 shows the control surface of the controller; this is based on the system knowledge and represents the output of the controller for any inputs combinations in the domain.

**Fig. 11 T1-IT2 absolute difference**

The T1-IT2 absolute difference shown in Fig. 11 for values between 0% and 6%, and this variation is proportional to the FOU and has an impact in the controller performance.

**Fig. 12 T1-GT2 absolute difference**

The T1-GT2 absolute difference is shown in Fig. 12 for values between 0% and 0.004%, and this variation is very little with respect to the T1-IT2 absolute difference.

**PERFORMANCE VARIATING THE FOU**

Fig. 13 shows the performance of the FLCs for plant 1, measured with the IAE metric and comparing the IT2 against GT2 without noise and in Fig. 14 with a 6.0206 dB of signal to noise ratio.
It is interesting to note that the performance behavior by varying the FOU of FLC, IT2 and GT2, we can observe a better performance behavior in GT2 FLC, as this controller shows a minor variation in control surfaces, however, this provides the controllers of a most stable performance.

**CONTROL PERFORMANCE METRICS**

Next the performance metrics obtained with the FLCs with a 26 dB SNR, are presented in Table III.

<table>
<thead>
<tr>
<th>Metric</th>
<th>T1</th>
<th>IT2</th>
<th>GT2</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSE</td>
<td>75.1324</td>
<td>78.849</td>
<td>76.6932</td>
</tr>
<tr>
<td>ISE</td>
<td>1.9462</td>
<td>1.9499</td>
<td>1.9464</td>
</tr>
<tr>
<td>IAE</td>
<td>2.9146</td>
<td>2.9277</td>
<td><strong>2.9039</strong></td>
</tr>
<tr>
<td>ITAE</td>
<td>7.2947</td>
<td>9.8286</td>
<td><strong>7.3934</strong></td>
</tr>
</tbody>
</table>

**E. D.C. motor speed controller**

The D.C. motor speed controller [27] is used as a common benchmark problem in many works, in different versions but is a stable second order plant in which the goal is reducing the error in the speed reference.

The FLC works with two inputs (Error and Error change) and with one output (Delta voltage) and this output is integrated in an analogous form to a PID Controller.

The difference between the T1 and T2 controllers can be better appreciated in Fig 18 and 19 with respect to IT2 and GT2 respectively.

The T1-IT2 absolute difference shown in Fig. 11 for values between 0% and 4%, and this variation is proportional to the FOU and has an impact in the controller performance.
The T1-GT2 absolute difference shown in Fig. 12 for values between 0% and 1%, and this variation is little with respect to the T1-IT2 absolute difference.

PERFORMANCE VARIATING THE FOU

Fig. 19 shows the performance of the FLCs for plant 1, measured with the IAE metric and comparing the IT2 against the GT2 without noise and in Fig. 20 with a 6.0206 dB of signal to noise ratio.

In Table IV we can note improvement in control performance with the Type-2 FLCs, on the other hand, IT2 shows similar performance to GT2 FLC, however, the performance of IT2 FLC tends to decrease when the FOU increases, while the GT2 FLC performance shows more stability.

IV. CONCLUSIONS

In conclusion, we consider two very different plants, and based on the control surface we can observe that the difference in both plants of an IT2 FLC and the GT2 FLC with respect to T1 FLC we have a similar behavior, this is, the difference is largest with respect to IT2 FLC than with GT2 FLC. Considering the performance when the FOU is changing we can observe a very irregular performance in the IT2 FLC, achieving in some conditions better results than the GT2 FLC, but the performance of GT2 FLC shows better results in most of the conditions of the FOU. The results reported in control performance metrics do not show great difference in using T2 against T1 in plant 1, but on the other hand, the results obtained in plant 2 are better with IT2 FLC and GT2 FLC. Based on the experiments and considering the applications we propose in Table V preliminary criteria to select a particular type of FLC in control applications.

As future work we will consider to evaluating the FLC by varying the MFs of the input-outputs fuzzy sets. In addition, include different metrics, such as stability metrics and implement the method with more complex problems.

References

[1] L. Amador-Angulo and O. Castillo, “Optimization of the Type-1 and Type-2 fuzzy controller design for the water tank using the
controller design using a new bee colony algorithm with fuzzy


