investigating the Topological Properties of C^{2}Torus Interconnection Network

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Abstract - *One of the important research effort was devoted to solve problems related to the connectivity and diameter of these topologies beside the routing algorithms. Recently, the C^{2}Torus interconnection network was proposed by Ram Krishna and et al to enhance the properties to Torus and Mesh via adding extra links in order to obtain lower diameter and shorter paths. The C^{2}Torus network shown to be more efficient and has better performance compared with the Torus and Mesh. The C^{2}Torus graph was presented without any deep details regarding its new topological property and its routing algorithm. To overcome the shortage issue in this problem, we are presenting in this paper a detailed investigation and formal research presentation of the new topology by defining the topological properties of the C^{2}Torus interconnection network including connectivity. Furthermore, we propose a new efficient routing algorithm for this attractive network.*

Keywords: *Interconnection Networks, Topological Properties, Network Connectivity, Diameter, Routing Algorithm.*

1 Introduction

A new efficient topology named C^{2}Torus interconnection network was proposed by Manish Bhardwaj and Parachi Chauhan [1]. It is based on the well-known two dimensional torus topology, and showed to have excellent physical connection by utilizing the center nodes. In this paper we give more effort and research for this new proposed topology and we study and analyze its topological properties in more details including size, connectivity, number of links, Node degree, bisection width, parallel paths. The routing algorithms are core issue in any topological properties and a lot of research efforts devoted to this issue [2]. An efficient routing algorithm is proposed for this topology in case there is only one center node when the size of the network is odd or when there are four center nodes when the network size is even.

This organization of this paper is follows: section 1 is the introduction. Section 2 investigates the related work of C^{2}Torus network, section 3 discusses and investigates the topological properties of C^{2}Torus network, section 4 proposes a new efficient routing algorithm for this network in case of odd and even sizes. Finally, section 5 concludes this paper.

2 Related Work

Many attractive interconnection networks topologies have been introduced to be attractive topologies for High speed Parallel computing (HSPC) including, star, arrangement star, OTIS-Mesh, OTIS-Arrangement star and OTIS-Cube [2 - 4]. Still the Torus and Mesh networks are well known networks in literature [5, 6], a lot of research efforts has been devoted for these networks but still the main problem the Mesh and Torus suffers from is that as their size increase the efficiency and the performance decrease dramatically furthermore the diameter of the mesh is still its main problem. As a proposed solution for these networks and overcome these problems the C^{2}Torus interconnection network based on two dimensional Torus was proposed in literature [1, 7]. The proposed network is suffering from the shortage of giving a formal definition, detailed topological property, and efficient and clear routing algorithm.

Due to the attractive properties of Torus, there were many research activities that utilized the Torus and presented more advanced interconnection networks developed by using the Torus as the base topology and construct it with extra hierarchy or engage it with another topology with different technological background; e.g. Optical connectivity networks; [8-13]. Similar research works have been done to a related family of interconnection networks with different base or different hierarchy [14, 15].

In this paper beside analyzing and studying the topological properties of C^2Torus network more deeply we propose a new efficient routing algorithm for the C^2Mesh Interconnection network topology based on 2D Mesh [6]. One of the considered issues related in to the C^2Torus interconnection network that the searchers have achieved the center nodes in case of odd or even sizes in a trivial way. In this paper we enhanced the topological properties also an important point regarding the routing algorithm which proposed in [7] it is a general one and it did not consider all the possibilities in transmitting the messages between the source and the destination work through the utilizing the center and corner nodes.

3 Preliminaries and Notations

The 2-D torus, Q_2^n, is defined as an undirected graph with n^2 nodes. Each node X is labeled in the form $X= x_1 ,x_0$, where $0 \le x_i < n$. Two nodes $X= x_1 ,x_0$ and $Y= y_1 ,y_0$ are neighbours through an edge *iff*, $\exists\ i$, $0 \le i < 2$, with condition satisfy $x_i = y_i \pm 1$ (remainder n) and $x_j = y_j$ for $i \ne j$ .We clear that we will ignore writing the reminder n for the rest of the statements in our paper. Q_2^n with degree 4 and it has a maximum distance (diameter) of n. Optimal path distance of the nodes X and Y equal to their *Lee distance* of them and is equal to

$$d_l(X,Y) = \sum_{i=0}^{1} w_i \text{ where}$$
$$w_i = \min_{0 \le i < 2}(|x_i - y_i|, n - |x_i - y_i|)$$

Figure 1: 2-D Torus, Q_2^3, that contains an 3^2 nodes, three in each row or column

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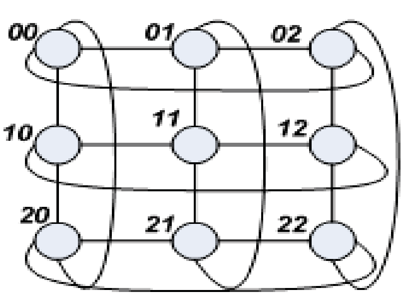

Figure 1: 2-D Torus, Q_2^3, that contains an 3^2 nodes, three in each row or column

The direct nodes of a node X, of nay i^{th} dimension are labelled by $X^{(i+)}$ and $X^{(i-)}$. It is clear that any node X has four neighbours, two direct neighbours on each dimension i, $0 \le i < 2$. The length of the Hamming *distance* from the nodes X to Y, denoted *HDIST* (X, Y) is defined as the total number of differences in their dimensions. A path between nodes X and Y is a best *or shortest path*, if it satisfy that the length is equal to $d_L(X,Y)$, this means that the path has the lowest distance from X and Y. such that $x_i \ne y_i$, a neighbour $X^{(i\pm)}$ is considered *preferred neighbour* of X for the routing from X to Y if $d_L(X^{(i\pm)}, Y)= d_L(X, Y) - 1$. In this case we could say that $i\pm$ is considered to be desirable *direction*. We may get the best and the shortest path through choosing the shortest direction we move for every taken step (dynamic). In case that $x_i \ne y_i$, a neighbour $X^{(i\pm)}$ and $d_L(X^{(i\pm)}, Y) \ge d_L(X, Y)$ is named a *spare neighbour. Note that the other n*eighbours different from preferred or spare are considered *disturb neighbours*. Now to route from a node X to a node Y, a disturb neighbour $X^{(i\pm)}$ of X corresponds to the situation where $x_i = y_i$ we may conclude that the i^{th} digit is disturbed. Travelling and routing using the disturb neighbour will raise the total routing elapsed two or more over the minimum distance. When routing using a spare neighbour will surely increase the elapsed routing distance one or more over the minimum distance. When we route between the two nodes X and Y, a node *TRANS* will be considered as a preferred *transit node* if $d_L(TRANS, Y) < d_L(X,Y)$. Figure 1 shows a 2-*D* torus; Q_2^n; where $n = 3$.

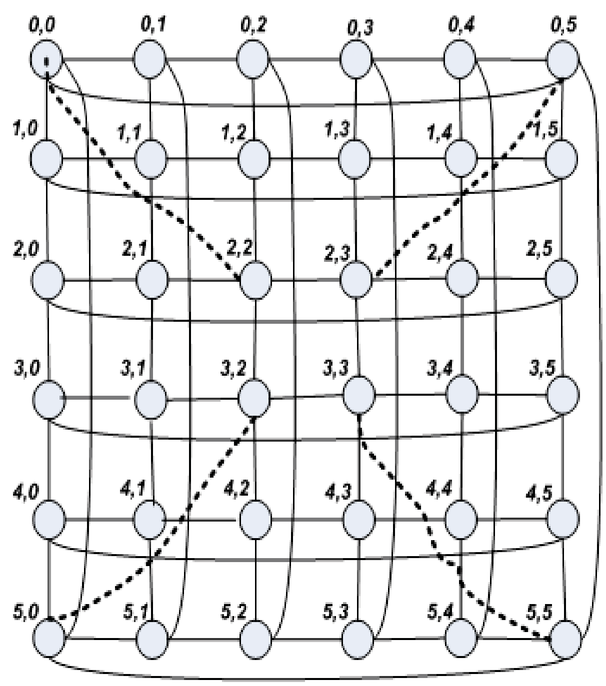

Figure 2: C^2Torus; $C2_2^n$; that contains an n^2 nodes with four center nodes

4 C2 Torus Topological Properties

C^2Torus; $C2_2^n$; is basically a 2D Torus that contains an n^2 nodes with extra four links that that connect the center of the Torus network with the four edge nodes of it; corner nodes. The center of the C2 Torus consists of one node if n is odd and consists of four nodes if the n is even as shown in figure 1 and figure 2 respectively.

Corner node: Any node is considered to be a corner node if and only if it belongs to the set of permutations (0, n-1).

Center node: Any node is considered to be a corned node if and only if it belongs to the set of permutations$\left(\left\lfloor\frac{n-1}{2}\right\rfloor, \left\lceil\frac{n-1}{2}\right\rceil\right)$.

The four extra links that connect the different corners with the center nodes of the C^2Torus must satisfy the fowling conditions:

If X is a corner node and Y is a center node, then

$$|x - y_i| = \left\lfloor\frac{n-1}{2}\right\rfloor \; for\ all\ 0 \leq i < n$$
$$|i - i| = n$$

Noting that when n is odd, and then there are 4 center nodes that connect the 4 corners of C^2Torus. While when n is even then there is only one center node that connects the 4 corners.

Example 1: In case that the size of C2 Torus is odd when n =7, it follows that its size is 49. The center code (3, 3) has four corner nodes connected to it, (0,0), (0,6), (6,0), and (6,6). On the other hand, when n = 6, then the C2 Torus size will be 36, hence we will have 4 center nodes connected to four corner nodes. The corner nodes (0,0), (0,5), (5,0), and (5,5) are connected to the center nodes (2,2), 2,3), (3,2), and (3,3) respectively.

Example 2: The routing path from a certain node to another when the size of C2 Torus is n =7 can be expressed as follows (random samples):

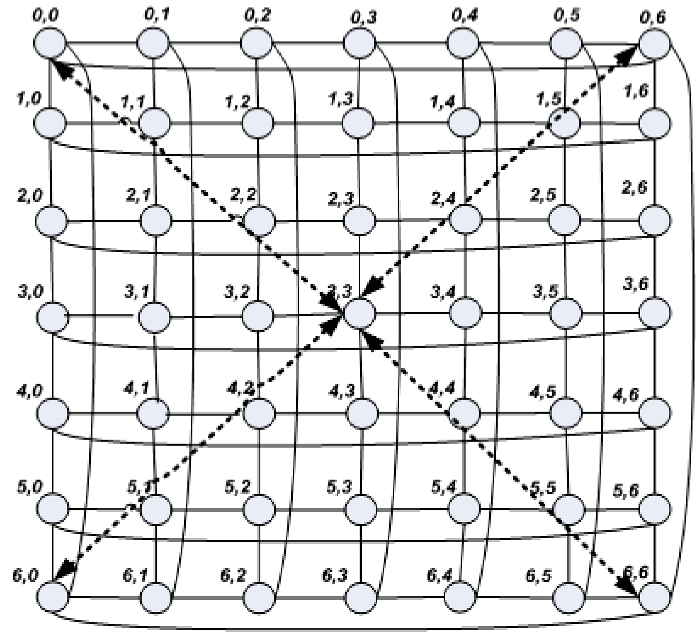

Fig3: C^2Torus; $C2_2^n$; contains n^2 nodes with one center node

The C^2Torus, $C2_2^n$, is an undirected graph with n^2 nodes. Each node X is labeled in the form $X= x_1, x_0$, where $0 \leq x_i < n$. Two nodes $X= x_1, x_0$ and $Y= y_1, y_0$ are joined by a link if, and only if, there exists i, $0 \leq i < 2$, such that $x_i = y_i \pm 1 \pmod{n}$ and $x_j = y_j$ for $i \neq j$ or $|x_i - y_i| = \left\lfloor\frac{n-1}{2}\right\rfloor \; for\ all\ 0 \leq i < n$. $C2_2^n$ has a degree of 4 and diameter of n-1. The *Lee distance* between x and Y nodes is given by

$$d_l(X,Y) = \sum_{i=0}^{1} w_i \quad \text{where}$$
$$w_i = \min_{0 \leq i < 2}(|x - y_i|, n - |x_i - y_i|)$$

The shortest path between nodes S and D is equal to

Where S, D, Co, Ce, are the Source node, Destination node, Corner node, and Center node consequently.

Two directed nodes of a node X through the i^{th} dimension are referred to as $X^{(i+)}$ and $X^{(i-)}$. So the Node X has at least four neighbours, two of these neighbours are on each dimension i, $0 \leq i < 2$. The corner node has an extra neighbours by connecting it with center node, and the center node has either one extra neighbour if n is odd, or four extra neighbours if n is even. When $x_i \neq y_i$, a neighbour $X^{(i\pm)}$. This direct node will be called a *preferred neighbour* of X for travelling from source node X to the node Y, if d_L ($X^{(i\pm)}$, Y)<

$d_L(X, Y)$. In this manner we conclude that $i\pm$ is a *preferred direction*. The optimal considered path can be issued by traversing on a preferred direction step move at every routing step. Keep in mind that in routing between the two nodes *X* to and *Y*, the node *TRANS* in this case is the most suitable choice. Node (preferred *transit node)* if $d_L(T, Y) < d_L(X, Y)$.

5 Routing Algorithm for C²Torus

In this section we will define and propose an efficient routing algorithm for the C2Torus interconnection network to transmit data from any source node S to any destination node D. We will utilize the attractive topological properties of C2Torus to find the shortest path between S and D. From the previous section we showed that the diameter of C2Torus is less than the diameter of Torus by one, but the effect of finding the shortest path for any two nodes within the network is much greater than the nodes at diameter distance. This can be clearly noticed when the two nodes are around the center and the corner nodes where the distance path is much shorter in C2Torus than it is in Torus due to the direct connection between center and corner nodes which make the routing distance shorter special with large network sizes.

```
Algorithm UV_Routing (M: message;S, D: node)
/* to route the message M to from Source Node S to its destination node D */
T=S  // transpose node at first = Source
While d_L (T, D) ≠ 0
{
        Let A^(i±) be the reachable preferred neighbour with least
    min (d_L (T, D), d_L(T, Co)+ d_L (D, Ce) + 1,
    d_L (T, Ce) + d_L
    {
  if min= d_L(T,D) then  T= A^(i±)  //direct preferred neighbour across a0 or ao
  if min= d_L(T,Co) + d_L(D,Ce) + 1 then T=Co
  if min= d_L(T,Co) + d_L(D,Ce) + 1 and d_L(T,Co) = 0 then T=Ce
  if min= d_L(T,Ce) + d_L(D,Co) + 1 then T=Ce
  if min= d_L(T,Ce) + d_L(D,Co) + 1 and d_L(T,Ce) = 0 then T=Co
      }
   then send M to T  // intermediate node
}
T = D then exit;  /* destination reached */
End.
```

6 Conclusions

We have presented in this research work an enhanced development on the topological properties of the well-known tours interconnection network to solve problems related to the connectivity and distance of paths within the topology. Recently, the C2Torus interconnection network was proposed by Ram Krishna and et al to enhance the properties to Torus via adding extra links in order to obtain lower diameter and shorter paths. The C2Torus graph was presented without any deep details regarding its new topological property and its routing algorithm. To overcome this shortage, we presented in this paper a detailed and more formal research study of the new topology by defining and analyzing the topological properties of the C2Torus interconnection network including connectivity. Furthermore, we proposed a new efficient routing algorithm to this attractive network.

7 References

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