Radar Waveform Design based on OFDM signals for Cognitive Radar Application

Vahid Karimi[1] and Reza Mohseni[2]

[1, 2] Digital Communications Signal Processing Research Lab., Department of Electrical Engineering, Shiraz University of Technology (SUTECH), Shiraz, Iran.
Author Emails: v.karimi@sutech.ac.ir; mohseni@sutech.ac.ir;

Abstract

A new orthogonal frequency division multiplexing (OFDM) radar waveform design method for cognitive radar is presented. The proposed waveform is designed based on maximizing mutual information (MI) criterion between the target impulse response and the received echoes given the knowledge of transmitted waveforms. Simulation results show that the proposed waveform design method achieves significantly higher rate of performance improvement compared with conventional waveform design methods.

Keywords: Cognitive radar, Mutual Information, Waveform Design, orthogonal frequency division multiplexing (OFDM).

Introduction

Cognitive radar is an innovative paradigm which is the combination of bio-inspired cognitive concepts and advanced system design [1]. All radar parameters are varied on a pulse-by-pulse basis in the current technology. This flexibility offers the opportunity to adopt approaches on both the transmitter and the receiver part [2- 4]. Cognitive radar has the ability to learn from experience on how to deal with different targets, large and small, and at widely varying ranges, all in an effective and robust manner. OFDM signaling scheme [5- 7], which is one way to use several subcarriers simultaneously is utilized to resolve and exploit the multipath components. It was shown in [8] that OFDM provided improved elevation coverage of surface-based radars in multipath conditions, but at the expense of increased transmit power.

We have proposed an algorithm to design the complex weights of OFDM signal based on the radar characteristic scene. Then mutual information criterion is applied to design the OFDM signal complex weights.

Mathematical statement of problem

An OFDM signal consists of L active subcarriers, a bandwidth of B *Hz*, and pulse duration of T seconds can be described by:

$$\mathrm{s}(t)=\sum_{l=0}^{L-1} a_l \exp(j2\pi l\Delta f t) \qquad (1)$$

Where $\Delta f = B/L+1 = 1/T$ denotes the subcarrier spacing and a_l represents the complex weights transmitted over the subcarriers. Let f_c be the carrier frequency of modulation, then the transmitted signal can be represented as:

$$\tilde{S}(\mathrm{t}) = 2\,\mathrm{Re}\left\{\sum_{l=0}^{L-1} a_l\, \mathrm{e}^{j2\pi f_l t}\right\} \qquad (2)$$

Where $f_l = f_c + l\Delta f \quad l = 0,1,\ldots,\mathrm{L}-1$. We have assumed extended target model in this paper and corresponding to a specific range cell containing the target, the received signal due to only the lth subcarrier is given by:

$$\tilde{y}_l(\mathrm{t}) = x_l \tilde{s}(\gamma(\mathrm{t}-\tau_l)) + \tilde{n}_l(\mathrm{t}) \qquad (3)$$

Where x_l are the complex weights representing the scattering coefficients of the target along the lth subchannel; $\gamma = 1+\beta$ where β is the relative Doppler shift; τ_l is the round-trip delay due to each scatterer points and c is the speed of propagation. $\tilde{n}_l(t)$ is the measurement noise along the lth subchannel. The complex envelope of the received signal at the output of the lth subchannel is:

$$y_{l_{OFDM}}(\mathrm{n}) = a_l x_l \phi_l(\mathrm{n}) + \mathrm{e}_l(\mathrm{n}) \qquad n = 0,1,\ldots,N-1 \qquad (4)$$

Where

$$\phi_l(\mathrm{n}) \triangleq e^{-j2\pi f_l \tau_0} e^{j2\pi f_l \beta_0 n T_{PRI}} \qquad (5)$$

After some simplification, the OFDM measurement model can be written as shown in equation (6).

$$Y_{OFDM} = A(\mathrm{a})\,\mathrm{X}\,\Phi(\nu) + \mathrm{E} = \mathrm{A}\,\tilde{\Phi} + \mathrm{E} \qquad (6)$$

Where $A(\mathrm{a}) \in \mathbb{C}^{L\times L}$ is complex diagonal matrix that contains the transmitted weights (a_l), $X \in \mathbb{C}^{L\times L}$ represents diagonal matrix containing the scattering coefficients of the target (x_l), $\Phi(\nu) \in \mathbb{C}^{L\times N}$ is a matrix containing the known Doppler information of the target through the target velocity parameter ν and $E \in \mathbb{C}^{L\times N}$ is the matrix of measurement noise.

MI criterion suggests the maximization of mutual information between the unknown channel and received signal as a function of transmitted training symbols. Mutual information between Y and $\tilde{\Phi}$ is given by:

$$\begin{aligned} MI(\mathrm{Y};\tilde{\Phi}\,|\,\mathrm{A}) &= \mathrm{h}(\mathrm{Y}\,|\,\mathrm{A}) - \mathrm{h}(\mathrm{Y}\,|\,\tilde{\Phi},\mathrm{A}) = \mathrm{h}(\mathrm{Y}\,|\,\mathrm{A}) - \mathrm{h}(\mathrm{E}) \\ &= N\left[\log\det\left(AR_{\tilde{\Phi}}A^H + R_n\right) - \log\det\left(R_n\right)\right] \end{aligned} \qquad (7)$$

Where *MI (x;y)* denotes mutual information between *x* and *y*, *h(x)* denotes the entropy of *x* and $R_x = E\left[xx^H\right]$ is the covariance matrix. To make the optimization problem nontrivial, we add the power constraint to this optimization problem and considering Additive white Gaussian noise, the optimization problem is stated as follows:

$$\begin{aligned} A_{opt} &= \underset{A}{\operatorname{argmax}}\left[\log\left(\det\left(\sigma^2 + AR_{\tilde{\Phi}}A^H\right)\right)\right] \\ &= \underset{\beta_l}{\operatorname{argmax}}\left[\sum_{l=1}^{L}\log\left(\sigma^2 + (N+1)\beta_l C_X(l,l)\right)\right] \qquad (8) \\ &\qquad s.\mathrm{t} \quad \sum_{l=1}^{L}\beta_l \le \mathrm{P}_t \end{aligned}$$

Where P_t is the total transmitted power and $\left[A_{opt}\right]_{l,l}=\sqrt{\beta_l}$ denotes the power transmitted along the *lth* subcarrier. The optimization problem can be solved by the method of Lagrange multipliers and the optimal waveform will be:

$$\beta_l=\max\left(-\frac{1}{\lambda}-\frac{\sigma^2}{(\mathrm{N}+1)\mathrm{C}_X(l,l)},0\right)\quad l=1,2,\ldots,L \tag{9}$$

Where the parameter λ is found by solving:

$$\sum_{l=1}^{L}\max\left(-\frac{1}{\lambda}-\frac{\sigma^2}{(\mathrm{N}+1)\mathrm{C}_X(l,l)},0\right)=P_t \tag{10}$$

Simulation Results

In this Section we simulate the waveform design algorithm proposed in this paper. We have considered stochastic extended targets with large number of scatterer points where located in the range cell centered at 3 km with respect to the radar (positioned at the origin). The scattering coefficients of the target (i.e., the entries of *X*) are unknown and generated from a *CN* (0, C_X) complex normal distribution where the target covariance matrix C_X is known. A conventional OFDM radar system (entitled in this paper as Equal-power waveform), in which their complex weights of the transmitted signal are given by $a_l=\sqrt{\frac{P_t}{L}}\ for\ l=1,2,\ldots,L$ in the first *N* pulse, will be considered to be compare with the proposed power allocation method.

Figure 1 shows mutual information of the proposed waveform and Equal-power for different total transmitted powers. It can be seen that the proposed waveform design technique results is significantly better performance compared with Equal-power waveform. It is obvious that both methods will have the same performance with increasing total transmitted power. The detection probability of the proposed waveform and Equal-power waveform is illustrated in Figure 2. As shown in this figure, the probability of target detection related to the proposed waveform is superior compare to Equal-power waveform.

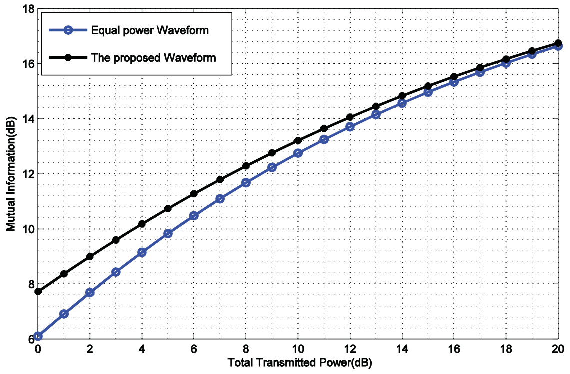

Figure1: Mutual information of the proposed power waveform compared with Equal-power waveform Vs. total transmitted power.

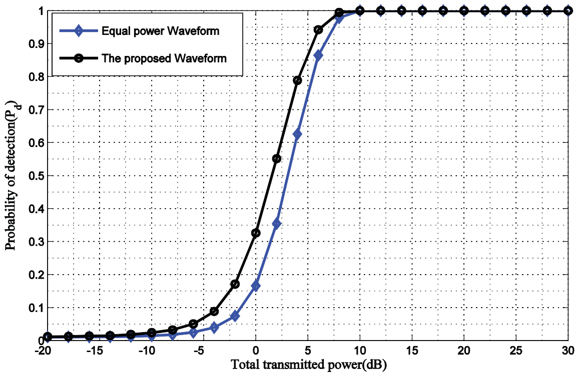

Figure2: Detection probability of the proposed power waveform compared with Equal-power waveform Vs. total transmitted power.

Conclusion

In this paper, we have proposed a new OFDM radar waveform design method applied in cognitive radar application. We have investigated signal design based on optimizing mutual information criterion. The results show that the proposed transmitted OFDM waveform will result in higher MI and better target detection performance, which is one of the main motivations of this work.

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